Synchronization and Self-Organization in Organizations
The Combinatory Systems View

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Abstract - In any organization people may be observed from two different points of view: first of all as organs (or components of organs), in the sense that organizational relationships assign them a precise spatial and temporal placement, they carry out a specialized function in relation to the entire structure, they have a specific functionality that delimits the admissible interactions with the other elements, and they show a particular functioning; secondly, as members of a social collectivity in which, independently of their organizational role and connections, they appear as similar agents, moved by private objectives, capable of producing micro decisions, behaviours and effects relatively analogous to that of the others.

My study considers the second aspect of human behaviour in organizations and aims to present a simple theory - the Theory of Combinatory Systems - which may allow us to describe, explain, and to a certain extent control many relevant and intriguing collective phenomena and their observable effects, which are produced by agents acting as unorganized social systems, even if they are also members of an organizational structure.

In plain words I define as a combinatory system any collectivity (composed of individuals or organizations) whose agents, consciously or unconsciously, act (exclusively or prevalently) on the basis of global information which they directly produce and update as the consequence of their micro behaviours.

On the one hand, the global information is - or derives from – a synthetic variable whose values are produced by the combination of the micro states of the agents (hence the name Combinatory System) but, on the other, these values affect the subsequent states as a result of a micro-macro feedback, acting over a period of time, that produces self-organization in the agents’ micro behaviours.

If we accept the traditional definition of self-organization as the macro behaviour of a collectivity of agents in which the micro behaviours appear to be “directed”, or “organized”, by an Invisible Hand, or Supreme Authority, in order to produce the emerging phenomenon represented by the formation of ordered structures, of recognizable patterns, then it is easy to recognize that the macro phenomena produced by the macro behaviour of the system become factors in self-organization, since they are interpreted by the agents as information they can base their decisions on.

In other words, the invisible hand is nothing other than the synergetic effect of the micro-macro feedback action (or circular causality) that generates and updates the global information that produces self-organization and emerging macro behaviours attributable to the collectivity.

On the basis of the previous definitions it is possible to understand some evident phenomena derived from combinatory systems composed of business organizations: the genesis of industrial clusters (systems of accumulation and diffusion); the spread of ideas or practices within organizations, or of a fashion in markets (diffusion systems); the continuous improvement of productivity and of quality levels (improvement and progress systems); path dependence and chaos in collective behaviour (irreversible and reversible stochastic diffusion systems).
In order to clarify the notion of combinatory system, as well as that of combinatory automaton as a logical instrument to formally simulate its behaviour, I feel it necessary to construct a conceptual framework of the different types of system.

Combinatory systems represent a particular class of dynamic or behavioural system that produces a dynamic process over time that is observed as the macro behaviour deriving from the micro behaviour of the structural elements making up the system.

The simplest way to consider dynamic systems is the synthetic or exogenous one, which is typically mathematical; this interprets these systems as black boxes and describes them through a system of differential equations, or as differences, that express the relations among the variations in inputs and those in outputs, considering the latter as at most the consequence of variations in the internal state variables. The nature of the elements that make up the structure or the constituent relations does not appear to be relevant for describing the system.

When instead the observer analyzes the nature of the elements that make up the structure of the system, he places himself in an endogenous perspective; this represents an analytical approach, which is typically logical, that considers systems as white boxes and tries to examine the nature of the internal elements and the relations that make up the structure in order to understand how these elements interact to produce the structural behaviours.

With regard to the analytical approach, we can distinguish between:

1. organized systems: these are characterized by elements linked by stable organizational relations – the organization, in other words – that specify, for each structural element, the following four elements: (i) a precise spatial and temporal placement (topology), (ii) a specialized function in relation to the entire structure, (iii) a specific functionality that delimits the admissible interactions with the other elements, (iv) a set of standards of functioning. Within organized systems we can, based on the autonomy of the structural elements, further distinguish between:

   1.1 organizations: the structural elements have no behaviour or autonomous significance except in relation to the higher organizational level; they are organs of the system linked to the organization and to the functionality of all the organization’s elements;

   1.2 orgonizations: the organs are holons: that is, units having decision-making and behavioural flexibility that are significantly autonomous if observed in isolation (or from a lower level), but that take on the significance of a component of the organization if considered from a higher level. They are at the same time individuals and organs; since they are organs-holons, we can call them orgons and define orgonization as an organization made up of orgons;

1 Wiener’s Cybernetics (Wiener, 1948; von Foerster, 1960; Haken, 1977; Kauffman, 1993) and, in particular, Evolutionary Cybernetics (Campbell, 1960; Gould, 2000), von Bertalanffy’s General System Theory (von Bertalanffy, 1968), Ashby’s System Theory (Ashby, 1956; Rapoport, 1984), and Klar’s System Science (Klar, 1991), Population Dynamics Models and Malthusian models and Volterra-Lokte equations in various forms (Volterra, 1931; Ardeni and Gallegati, 1999) and the autopoietic approach (Maturana and Varela, 1980; Varela, 1981), offer a rich set of mathematical tools to describe, from an exogenous point of view, the behaviour of collectivities when viewed as black boxes which produce multiple responses to multiple stimuli conditioned by multiple external feedback. Internal feedback among organs or agents is by definition unobservable (or, in any event, not relevant for the description of the system) (Sandquist, 1985: 22).

2 Holons form Holararchies, defined as a hierarchically organized structure of holons. In a Holarity each Holon could be regarded as either a whole or as a part, depending on how one looks at it. A Holon will look as a whole to those parts beneath it in the hierarchy, but it will look as a part to the wholes above it. So, a Holarity is then a whole that is also a structure of parts that are in themselves wholes. Nevertheless Holararchies must not be confused with Orgonizations, since in the latter the holons not only have a significance relative to the level of observation, but also relative to the organizational relations with the other holons that transform them into organs in the organized structure of the system.
2. non-organized systems or social collectivities: there are no stable organized relations between component elements that are all at the same level without being necessarily interconnected by evident interactions, or by network, web or tree structures; these are individual agents (and can be composed of mono-level holons) which can be uniform or differentiated with respect to their nature or individual behaviour; for this reason these systems can be succinctly denominated Agent-Based Systems. If observed from a certain distance collectivities appear distinct with respect to their component elements, and thus seem able to produce an autonomous macro behaviour (and at times a macro effect or a recognizable pattern) which is not included in advance in the operating programme of the agents’ behaviour. Each agent produces its own micro behaviour; the macro behaviour of the system appears as an emerging characteristic that derives from the individual micro behaviours. With regard to non-organized systems we can further distinguish between:

2.1 complex (adaptive) systems: these have two basic characteristics: the agents are normally divided into classes, and they interact according to local rules that establish how the micro behaviour of an agent derives from that of its neighbours;

2.2 combinatorial systems: these represent a particular class of unorganized system made up of a collectivity of similar agents (not functionally specialized and not necessarily interconnected by evident interactions) each of which is capable of producing a micro behaviour, and a macro effect, analogous to that of the others. If, on the one hand, the macro behaviour of the System, as a whole, derives from the combination – appropriately specified (sum, product, average, min, max, etc.) – of the analogous behaviours (or effects) of its similar agents (hence the name **Combinatory System**), on the other the macro behaviour (or the macro effect) determines, or conditions, or directs, by necessity, the subsequent micro behaviours (Fig. 1).

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1 Koestler’s holonic systems approach represents a different approach with respect to Agent-Based Systems (Koestler, 1968; Shimizu, 1987; Wilber, 2000), particularly useful for studying the behaviour of living organisms and social organizations. These are composed of self-reliant units that are capable of flexible behaviour. More specifically, though, a holon can be thought of as a special type of agent that is characteristically autonomous, cooperative and recursive, and that populates a system or a collectivity.

4 In Agent-Based Models collectivities are normally interpreted as Complex (Adaptive) Systems or CAS (Coveney and Highfield, 1995; Mitleton and Kelly, 1997; Allen, 1997; Axelrod, 1997; Goldspink, 2000), defined as a plurality (usually large) of blind (reactive) or intelligent (active) multi-character (Droogoul and Ferber, 1994), specialized, usually (strongly) interconnected (Wu, 1997; Granovetter, 1974; Grimmert, 1999) interacting agents (or processes) (Holland, 1995; Gell-Mann, 1995-96; Stacey, 1999), often showing possible multi-level hierarchies (Chan, 1998; Gaffeo, 1999; Cummings and Stewart, 1985) whose collective macro behaviour is determined by the interaction of the micro behaviours of the agents (Otter, Veen and Vriend, 2001) on the basis of simple local rules (Waldrop, 1993) according to a schema (innate or learned) (di Primio, 1999), and which show non-linear dynamics as well as unanticipated global properties, or patterns (Foster and Metcalfe, 2001: 4).

5 Combinatory systems differ from complex systems and, in particular, from CAS and from Holarchies in many aspects. Firstly, because combinatorial systems do not necessarily present phenomena of adaptation but, generally, some form of self-organization due to the micro-macro feedback, that is the adaptation of agents to a synthetic variable produced by the macro behaviour of the system. Adaptation may be a characteristic of some particular class of CS representing populations and not, in general, of collectivities conceived in a broader sense. A second difference is observable also as regards the similarity of the agents: “Here we confront directly the issues, and the questions, that distinguish CAS from other kinds of systems. One of the most obvious of these distinctions is the diversity of the agents that form CAS. Is this diversity the product of similar mechanisms in different CAS? Another distinction is more subtle, though equally pervasive and important. The interactions of agents in CAS is governed by anticipations engendered by learning and long-term adaptation.” (Holland, 1995: 93). The third main difference regards the absence of interactions among the agents; in combinatorial systems agents generally interact only with some macro variable and not with each other. The fourth relevant difference is that the theory of CAS observes the macro effects of the system produced by the agents that follow a schema or change the schema previously followed. Any micro-macro feedback between the micro behaviours and the schema is considered as a relevant characteristic. Finally, ignoring the micro-macro feedback implies that CAS theory only focuses its attention on necessitating factors and ignores the recombining ones. For a synthesis, see Table 1.
In this study I propose to show how organized systems and combinatory systems can interact; that is, how in organizations it is also possible for individuals, even if they belong to organs, to act as agents of combinatory systems and produce interesting phenomena typical of non-organized collectivities.

![Figure 1 - The operative logic of a combinatory system](image)

### 2 – The central idea of Combinatory System Theory (CST)

Social collectivities have always been a very complex subject of study, and for this reason both fascinating and interesting.

Conway’s discovery of the fantastic world of *Life* (Gardner, 1970), Schelling's (1971) model of neighbourhood segregation and Sakoda's (1971) model of group formation are fundamental milestones in the study and the simulation of the behaviour of collectivities.

The following, as the much famous *Micromotives and macrobehaviour* by Schelling (1978), is an attempt to offer a logical explanation for the collective macro behaviours shown by intelligent agents which, acting on behalf of their own interest, produce emergent collective dynamics.

<table>
<thead>
<tr>
<th>Table 1 - How do Combinatory Systems differ from Complex Systems?</th>
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<tbody>
<tr>
<td><strong>Complex systems and Holarchies</strong></td>
</tr>
<tr>
<td>Agents are heterogeneous</td>
</tr>
<tr>
<td>Agents are interconnected and show hierarchy</td>
</tr>
<tr>
<td>Micro behaviours are differentiated</td>
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<tr>
<td>Agents act following local rules</td>
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<tr>
<td>Decisions are prevalently based on the prisoner’s dilemma schema</td>
</tr>
</tbody>
</table>
It is rather difficult to provide a list of theories, models and instruments that can be used to explore this research field; we can nevertheless recognize two main approaches:

a. the *macro* or *analytic* approaches, which aim to build models of systems capable above all of justifying the *macro* behaviour; the *micro* behaviours are considered unobservable or not important because the relations that link the elements are too complex and numerous; the abundance of connections make the construction of meaningful models based on elements very difficult; so the macro approaches produce a macro description of the behaviour of collectivities;

b. the *micro* or *synthetic* approaches whose models are built exclusively (or prevalently) by studying the micro behaviours and the micro rules which connect them (Gilbert, 1995). The macro behaviour is a consequence – often unexpected – of the action of these connections. Forming part of this typology are the models worked out by the Cellular Automata Theory, which allow us to explore the systems by simulating Artificial Life.6.

A third approach for exploring collectivities is provided by the simple theory that I have called Combinatory System Theory, since it deals with a particular class of systems acting in a “combinatory” way.

The combinatorial systems approach is *neither* a macro approach, since it also refers to local rules considering micro behaviours, nor a micro approach, since it also includes the macro behaviour in the model of the system.

It is rather a micro-macro approach, precisely in that the operating rules, describing the behaviour of the system, must in some way include not only local rules but also the feedback between the micro and macro behaviours.7

The feedback arises from necessitating factors, which force the agents to adapt their micro behaviour to the system's macro behaviour, and is maintained by the action of recombining factors, which lead the collectivity to recombine the micro behaviours, or the micro effects, in order to produce and maintain the macro behaviour, or the macro effect.

Recognizing the existence of a micro-macro feedback and understanding the nature of both the necessitating factors and the recombining ones is indispensable for interpreting collective phenomena as deriving from a combinatory system.3

The Theory of Combinatory Systems searches for the conditions that produce the macro behaviours and proposes models to interpret the collective phenomenon. In particular, the theory focuses on the necessity of understanding the nature of the macro rules, which specify the recombining factor(s), and of the micro rules, which specify the necessitating factor(s); the joint action of these factors gives rise to and maintains the macro and micro behaviours.

The Theory also considers reversible systems (Lustick 2000) that have a cyclical behaviour and, under certain conditions concerning the probability function regarding the transition of state of the agents, a chaotic one as well (Gleick 1988, Kellert 1993).

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6 The micro approaches assume that the behaviour of collectivities is determined by local rules (Waldrop, 1993) - as in the traditional complex systems approach (Coveney and Highfield, 1995) and its related specific topics and tools: adaptive complex systems (Allen, 1997; Goldspink, 2000), cellular automata (Bak, 1994; Schatten, 1999), Alife approaches (http://alife.org/index.php?page=alife&context=alife and http://alife.santafe.edu/), such as Dorigo's Ants approach (Dorigo,1999; Hölldobler and Wilson, 1990), Langton’s Swarm approach (http://www.swarm.org/intro.html), Reynolds’s boids (Reynolds, 1987) and Dolan’s Floyds approach (Dolan, 1998). As Holland attempts to demonstrate, the most powerful approach to understanding and showing the hidden order in collective behaviour is the genetic algorithms approach (Holland, 1975, 1995) and the related genetic programming approach of Koza (Goldberg, 1989; Koza, 1992).

7 In complex systems theory the feedback is considered between agents and not as a determining feature of the system. See: http://pscs.physics.lsa.umich.edu/complexity.html, and http://home.online.no/~bergar/mazega.htm.

3 In order to provide a technical explanation of the action of such systems, and above all for the purpose of planning them, knowledge of the energy inputs can turn out to be indispensable.
In this sense *path dependence* (Arthur, 1988; Liebowitz and Margolis, 1998) is proof of the action of the micro-macro feedback, even if path dependence theory does not explicitly include this mechanism in the explanation of the path dependence.

Combinatory systems are *recursively closed systems*; their dynamics are prevalently due to the joint action of "chance" and "necessity"; they might thus also be called "chance-necessity" systems.\(^{10}\)

Other relevant characteristics (I will only mention these) concern the fact that, even though combinatorial systems are unorganized and closed systems, they can organize themselves into specialized subsystems and show ramifications (Monod, 1971; Maturana and Varela, 1987), and can expand their effects on elements belonging to a vaster environment.

If the micro behaviours of the agents are determined exclusively by the macro behaviour, the combinatorial system is a *pure combinatorial system*.

If they depend also on an opportune neighborhood as well as, naturally, on the macro behaviour, the combinatorial system is characterized by incomplete and limited information.

Finally, if the agents’ behaviour depends only on local rules acting on a defined neighborhood, without considering any micro-macro feedback, the system is a complex system and loses the characteristics of a combinatorial system and can be simulated by traditional *cellular automata*.\(^{12}\)

Three aspects of this theory make it particularly effective:

1 - it is not limited to describing the macro behaviour of the unit based on general rules or the individual behaviours based only on local rules, but tries to uncover and explain above all the *feedback* between the macro and micro behaviours or their effects;

2 - to understand the phenomena attributable to the action of combinatorial systems the theory tries to uncover and make clear the *necessitating factors* (that cause the micro behaviour of each agent in the system) and the *recombining factors* (that produce and maintain the unit’s macro behaviour). The theory then concludes that, in the presence of suitable necessitating and recombining factors, “chance” will trigger the dynamic process of the system that “by necessity” is then maintained and influences the individual behaviours;

\(^{10}\) We have used, though with a different meaning, the same terminology used by Monod (1971), who, in his famous *Chance and Necessity*, examined a very powerful combinatorial system: that leading to a dynamic evolution in a population due to random mutations produced in the DNA that "by necessity" spread as a result of the invariant reproductive mechanism of cells.

Haken also speaks of *chance* and *necessity* when he proposes constructing models of complex systems. Here Haken considers *chance* as the unpredictable fluctuation from an unstable equilibrium state, and *necessity* as the movement towards a new, more stable state (Haken, 1983; Prigogine and Stengers, 1984).

Chance will not only set under way the macro behaviour but will also determine the direction, that is the direction of the "winning" fluctuation. Prigogine bases his theory on the emergence of order in complex systems on the consequences of fluctuations (Nicolis and Prigogine, 1989; Haken 1983).

A simple way to observe the influence of the random fluctuations in orientating the direction of the "macro" dynamics of combinatorial systems - even if it is not sufficient to describe the effect of *chance* on the overall dynamics of a combinatorial system - is offered by the *Polya Urns* and by the *Ehrenfest Urns*.

\(^{12}\) For this reason we cannot in general consider the ants, the swarm and, more generally, the cellular automata approaches as examples of combinatorial systems, except in the case where the macro behaviour may affect the micro behaviour. This is the case of populations of insects, typically ants, which act by creating an “aromatic potential field” by spreading *pheromones* or other permanent messages. With their micro behaviours the agents spread pheromone across one site (micro information); the increasing concentration of pheromone (global or macro information) increases the probability that each agent will move in the direction of that site. The micro-macro feedback is quite evident (Zollo, Iandoli and De Maio, 2001; Deneubourg and Goss, 1989). This behaviour is the consequence of *stigmergy* (Grassé, 1959).
3 - the procedural explanation offered by the theory not only allows us to understand the operating mechanism that produces the phenomena under examination, but also permits us to determine the most effective forms of control.

3 – The formal definition of social combinatory system

The most interesting combinatory systems are the social ones, which are made up of men or organizations which, consciously or unconsciously, act (exclusively or prevalently) on the basis of global information which they directly produce and update as the consequence of their micro behaviours, following these functioning rules that also define a combinatory automaton (Fig. 2):

− the system is composed of a set of N agents \( A_i \), for \( 1 \leq i \leq N \);
− all agents are similar in the sense they show a relatively similar nature, structure or significance;
− the agents are not necessarily interconnected by evident interactions, or by network, web or tree structures;
− all the agents are characterized by the same individual variable (or set of variables) – \( a_i(t_h) \) – of some kind (qualitative or quantitative) whose values – at any time \( t_h \) – represent the individual states whose dynamics, over a period \( T \), may be defined as the micro behaviours of the agent – which may lead to analogous micro effects, \( e_i(t_h) \);
− the collectivity is characterized by a macro (global) variable (qualitative or quantitative) whose values – \( X(A, t_h) = \{C_{i \leq N} a_i(t_h)\} \) – at any time \( t_h \) represent the system states whose time series over a period \( T \) may be defined as a macro behaviour (which may lead to a macro effect of some kind, \( E(A, t_h) = F \{X(A, t_h)\} \) that may be conceived as the output of the collectivity as a whole;
− the system state – at any time \( t_h \) – derives from the combination \( C_{i \leq N} \) (to be specified) of the individual states, following macro or recombining rules, and may be conceived – or interpreted – as internal global information for the agents; in many cases the internal global information corresponds to the macro behaviour or the macro effect of the collectivity as a whole;
− through the global information each agent – at time \( t_{h+1} \) – can perceive and evaluate – in a simple pay-off table – positive or negative gaps (advantages or disadvantages) between his individual state and the state of the collectivity; following the micro or necessitating rules each agent makes individual micro decisions (by a process of imitation and social learning) in order to increase (if positive) or reduce (if negative) the perceived gaps; these micro decisions produce the transition of state of each agent as a function of both the previous micro state and the macro state, according to necessitating rules \( \mathbb{N}_i : a_i(t_{h+1}) = \mathbb{N}_i [a_i(t_h), X(A, t_h)] \);
− but these decisions recursively change the value assumed by the macro variable, and this modifies the perceived positive or negative gaps, driving the agents to adapt their behaviour by new decisions;
− for the recursive dynamics being produced, we must also assume that the initial state \( a_i(t_0) \) is specified.
The definition is summarized in the following formal model (Fig. 2):

\[
\begin{align*}
A(t_0) &= [a_i(t_0)] \leftarrow \text{CHANCE/PROGRAMME} & 1 \leq i \leq N \\
X(A, t_h) &= \mathcal{C}_{1 \leq i \leq N} [a_i(t_h)] = \mathcal{C} [A(t_h)] & h=0, 1, 2, \ldots \\
E(A, t_h) &= F \{ X(A, t_h) \} \\
a_i(t_{h+1}) &= \mathcal{N} [a_i(t_h), p_i, \Delta t_i, E(A, t_h)] \\
e_i(t_{h+1}) &= f_i \{ \mathcal{N} [a_i(t_h), p_i, \Delta t_i, X(A, t_h)] \} & 1 \leq i \leq N \\
\text{Set: } \{ \mathcal{C}_{1 \leq i \leq N}, \mathcal{N}_i, p_i, \Delta t_i, F \text{ and } f_i \}
\end{align*}
\]

Initial micro states: \( a_i(t_0) \)

Input due to CHANCE

MACRO CONTROL

Strengthening or weakening the recombining factors

RECOMBINING FACTORS

\( \mathcal{C}_{1 \leq i \leq N} \)

Lead the system to recombine the micro behaviours

MACRO FEEDBACK

SYNTHETIC STATE

\[ X(A, t_h) = \mathcal{C}_{1 \leq i \leq N} [a_i(t_h)] \]

Output due to NECESSITY

NECESSITATING FACTORS

\( \mathcal{N}_i \)

Force the agents to adapt their micro behaviours

MICRO CONTROL

Strengthening or weakening the necessitating factors

Micro effects \( e_i(T) \)

Micro behaviours \( a_i(T) \)

SINGLE AGENT \( a_i(t_h) \)

Figure 2 - The main elements of a combinatory system

The operative logic of combinatory systems is as basic as their structure (Fig. 3):

- on the one hand, the internal global information is - derives from or is associated with – the macro behaviour of the system, which may be thought of as an organizing or driving variable; thus the micro behaviours seem self-synchronized and self-organized to produce that macro behaviour which, for an observer, may be conceived as an emergent phenomenon;

- on the other hand, the macro behaviour updates the global information and determines, conditions, directs, or drives (together with external information, if present) the subsequent micro behaviours in a typical micro-macro feedback; this, for an observer, may be conceived as a self-organization effect;

- the micro-macro feedback operates between the limits of the minimum activation number and the maximum saturation number of the agents presenting the state that maintains the micro-macro feedback; this guarantees over time both the production of the emergent phenomenon and the maintenance of the self-organization effect.
4 – Self-organization and Synchronization in combinatory systems

The previous definition and typology emphasizes the cognitive activity of the agents in combinatory systems: the macro effects produced by the macro behaviour of the system in themselves do not necessarily lead to self-organization; they become factors in self-organization only when these effects are interpreted by the agents as information they can base their decisions on.

If we accept the traditional notion of self-organization as the macro behaviour of a collectivity of agents in which the micro behaviours appear to be directed, or organized, by an Invisible Hand, Supreme Authority, or Benevolent Deity in order to produce the emerging phenomenon represented by the formation of ordered structures, of recognizable patterns (Foster and Metcalfe, 2001: 130; Pelikan, 2001), then all the above-mentioned collective phenomena can also be defined as self-organization or spontaneous order (Sugden, 1989; Kauffman, 1993; Ashford, 1999; Swenson, 2000)\(^8\).

There is nothing strange here: the invisible hand is nothing other than the synergetic effect of the micro-macro feedback action (or circular causality) that generates and updates the global information that produces self-organization and emerging macro behaviours attributable to the collectivity.

The micro-macro feedback may be thought of as an internal dynamic director or, better yet, as an internal dynamic organizer which produces and uses the global information as an ordering

\(^8\) Adam Smith’s invisible hand naturally comes to mind. Adam Smith used the term “invisible hand” only once in his Wealth of Nations (1776) in the following quotation: “...[B]y directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention. Nor is it always the worse for the society that it was not part of it.”

The invisible hand was also mentioned by Haken, the founder of Synergetics: “We find that the various parts are arranged as if guided by an invisible hand and, on the other hand, it is the individual systems themselves that in turn create this invisible hand by means of the coordinated effect. We shall call this invisible hand that gives order to everything the «organizer” (Haken, 1977).
parameter\(^9\) and, following the *slaving principle*, directs or organizes the individual behaviours and produces the self-organization of the system and hence the collective phenomena (von Foerster, 1960, Haken; 1977, Prigogine, 1985; Nicolis and Prigogine, 1989; Kauffman, 1993).

Since by definition in combinatory systems the agents are similar and have similar behaviour, it follows that we can assume that the same information produces similar decisions regarding the change in state of the agents, who thus appear to conform or even *synchronize* their micro behaviours.

### 5 – Probabilistic social combinatory systems

The *social combinatory systems* that are most interesting and easiest to represent are the *irreversible* ones (build a tower or not, teach Italian or English to babies). In these systems both the micro and macro behaviours produce permanent effects that may be viewed as increasing or decreasing cumulative processes in which the probabilities are: \(p(X, t_0)_{[0,1]}\).

Chaos arises in *combinatory systems* when the *hypothesis of reversibility* is introduced (for example: to speak or to keep quiet in the next minute, wear a skirt or miniskirt on different days, choose road A or B on different days) (Fuchs and Haken, 1989). These systems are generally governed by *transition* probabilities: \(p(X, t_0)_{[-1,1]}\).

When reversibility in micro behaviours or in micro effects is possible, the combinatory system’s macro behaviour, or macro effect, can show a cyclical dynamic and, under certain conditions concerning the probability function regarding the transition of state of the elements, a chaotic one as well, when no cycles are recognizable in the time series of the system starting from random initial values (Gleick, 1988). Examples of reversible systems are those of diffusion and dissemination (fashion and contagion), whose elements may at different times present the same state chosen from a repertoire (Lustick, 2000).

In particular we can note that in probabilistic reversible combinatory systems both the random initial states of the system and the probability function for the transition of states, which depend on the macro behaviour at each iteration, can be determined with ample approximation.

These hypotheses of randomness in the initial conditions and in their evolution as well (history dependence), together with the imprecision of the measurement of the micro behaviours, produce dynamic instability in the macro behaviour and explain almost all the cases of path dependence, both in reversible and, in many cases, irreversible systems, as we can argue from [A.1] in the previous models (Liebowitz and Margolis, 1998; Arthur, 1988, 1990, 1994).

### 6 – Typology of Combinatory Systems. A short survey

The logic proposed in the previous sections can be observed in four relevant classes of combinatory systems which differ with regard to their macro behaviour (or their macro effect) (Fig. 3).

1 - Systems of *accumulation*, whose macro behaviour leads to a macro effect which is perceived as the accumulation or the clustering of “objects”, behaviours, or effects of some kind; this logic applies to quite a diverse range of phenomena, among which the formation of urban or industrial

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\(^9\) When an ordering parameter guides system components or subsystems, this is said to *slave* the subsystems, and this *slaving principle* is the key to understanding self-organizing systems. The *micro-macro feedback* is the expression of the *circular causality* which emerges when the subsystems collectively determine the order parameters and the order parameters determine the behavior of the subsystems (Haken, 1988).
settlements of the same kind and of industrial districts, the grouping of stores of the same type in the same street, the accumulation of garbage, graffiti, writings on walls; but it can also be applied to phenomena such as the breaking out of applause, the formation and the maintenance of colonies, forests, herds and schools.

The following heuristic model can describe these systems (Fig. 3.1):

**NECESSITATING RULE:** if you have to accumulate some object with others similar in nature (micro behaviour), look for already-made accumulations, since this gives you an advantage or reduces some disadvantage (necessitating factor);

**RECOMBINING RULE:** the environment preserves the accumulated objects or is not able to eliminate them, and maintains the advantages of the accumulation; everyone accumulates (macro behaviour) and an accumulation of some kind is created (macro effect);

**MICRO-MACRO FEEDBACK:** the larger the accumulation (macro effect), the more incentive (facility, probability) there is to accumulate (micro behaviours) objects (micro effects); the collective accumulation (macro behaviour) leads to the maintenance or the increase of the accumulation.

**Figure 3 – Typology of combinatory systems**

2 - **Systems of DIFFUSION**, whose macro effect is the diffusion of a trait or particularity, or of a “behaviour” or "state", from a limited number to a higher number of agents of the system; systems of diffusion explain quite a diverse range of phenomena: from the spread of a fashion and the appearance of artifacts and of furniture of the same kind and/or quality to that of epidemics and drugs; from the appearance of monuments of the same kind in the same place (the Towers of Pavia, for example, to the spread and maintenance of a mother tongue, or of customs; from the endogenous formation of a culture or mentality or custom among organs and people in organizations to the spread of an optimistic or pessimistic mood in the whole organization.

Heuristic model (Fig. 3.2):

**NECESSITATING RULE:** if you see that an "object" is diffused, then it is "useful" for you to possess it or harmful not to possess it (necessitating factor), and you must try to acquire it;
RECOMBINING RULE: the environment or the collectivity preserves the diffused objects and maintains the utility of possessing the object; the higher the utility or need to acquire the object for the individuals, the more the object will spread throughout the collectivity;

MICRO-MACRO FEEDBACK: a greater diffusion (macro effect) implies a greater desire to acquire the object (micro effect); the single acquisition (micro behaviour) widens the collective diffusion (macro behaviour).

3 - Systems of PURSUIT produce a behaviour that consists in a gradual shifting of the system toward an “objective”, as if the system, as a single entity, were pursuing a goal or trying to move toward increasingly more advanced states; this model can represent a lot of different combinatory systems: from the pursuit of records of all kinds to the formation of a buzzing in crowded locales; from the start of feuds and tribal wars in all ages to the overcoming of various types of limits of practices or procedures or rules; from the spread of a positive competitive atmosphere among organs or people to the spread of a negative competitive behaviour in organizations.

Heuristic model (Fig. 3.3):

NECESSITATING RULE: if there is an objective, try to achieve it; if there is a limit, try to exceed it; if another individual overtakes you (negative gap), regain the lost ground; if you're even with someone, try to go ahead; if you're in the lead, try to maintain or increase your advantage (positive gap);

RECOMBINING RULE: the collectivity recognizes the validity of the object and views limits in a negative way; the more individuals try to exceed the limit, the greater the chance of exceeding it, with a consequent advantage for those who succeed in doing so. This provides the incentive for the pursuit;

MICRO-MACRO FEEDBACK: if everyone tries to go beyond the limit (macro behaviour), then this is raised (macro effect), thereby eliminating the advantage for those who have already reached it (micro effect); this forces the individuals to exceed the limit (micro behaviour).

4 - Systems of ORDER, produce a macro behaviour, or a macro effect, perceived as the attainment and maintenance of an ordered arrangement among the agents that form the system; systems of order can be used to interpret a large number of phenomena: from the spontaneous formation of ordered dynamics (for an observer) in crowded places (dance halls, pools, city streets, etc.) to that of groups that proceed in a unified manner (herds in flight, flocks of birds, crowds, etc.); from the creation of paths in fields, of wheel-ruts on paved roads, of successions of holes in unpaved roads, to the ordered, and often artificial, arrangement of individuals (stadium wave, Can-Can dancers, Macedonian phalanx); from the spontaneous formation of the best or the worst practices to the acceptance of the same vision or mission or strategic guidelines. Heuristic model (Fig. 3.4):

NECESSITATING RULE: there are advantages in maintaining a particular order and disadvantages in breaking it; if you want to gain the advantages or avoid the disadvantages, try to adjust your behaviour so that you maintain or achieve the order that is indicated by the rules that establish it;

RECOMBINING RULE: the more the particular order is maintained, the greater the advantages from adjusting one's behaviour to maintain it and the disadvantages from breaking it;

MICRO-MACRO FEEDBACK: the order (macro effect) creates the convenience for individuals to maintain the arrangement and respect the rules (micro behaviours); everyone maintains a coordinated behaviour (macro behaviour).
7 - Systems of improvement and progress,

A very special and important combinatory system is the one I have named the *Improvement and Progress Combinatory System*, since its particular effect is to produce progress, understood as an improvement in the overall state of a collectivity that is attained through individual improvement.

*Figure 4 – Three types of systems of improvement and progress*

Table 1 – Improvement and Progress Combinatory Systems (10 agents, 10 iterations)

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<td>Initial states</td>
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<tr>
<td>Parameters</td>
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1.(B) – Simulations (red lines indicates the progress variable; coloured lines indicate agents’ improvement variable)
These systems can be classified among those belonging to the classes mentioned above; in particular they are systems of pursuit that produce accumulation or diffusion; I shall describe them as an independent class only because of their particular relevance in social collectivities.

Individual improvements raise the parameter that measures collective progress; this constitutes the global information that leads to the perception of positive and negative gaps that push the individuals to improve in order to increase the gaps (if positive) or eliminate them (if negative) (Fig. 3.5).

The system must be able to notice the individual improvement and to adjust the progress parameter to the average (or, more generally, to the combination) of the individual improvement measures.

Among the phenomena that can be explained using the system of improvement and progress are the growth of productivity in firms, the continuous improvement in the quality of products, progress in the sciences and in technology, and the evolution of all types of species as a consequence of individual choices. Heuristic model:

**NECESSITATING RULE:** if you perceive that the level of your improvement parameter is below the level of the system's progress parameter – that is, that there is a negative gap between your state and that of the others – try to improve in order to reduce the gap and, if possible, try to attain a positive gap; if you perceive there is a positive gap, do nothing or try to improve further in order to increase the favorable gap;

**RECOMBINING RULE:** the system must be able to notice the individual improvement and adjust the progress parameter to the average (or, more generally, to the combination) of the individual improvement measures;

**MICRO-MACRO FEEDBACK:** individual improvement (micro effect) raises the parameter that measures collective progress (macro effect); this leads to the formation of positive and negative gaps that push the individuals to improve in order to increase the gaps (if positive) or eliminate them (if negative).

When “by chance” or “by programme” an improvement begins in one or all of the agents of the system, then “by necessity” progress occurs throughout the system; the improvement spreads and the progress continues, unless a limiting state is reached in which no further improvement can be carried out and no further progress can occur.  

The general model representing these systems is:

\[
\begin{align*}
\mathbf{\mu}(t_0) & \leftarrow \text{« CHANCE/PROGRAMME »} \\
\pi(A, t_0) & = F \{ \mathbf{C}_{1 \leq n \leq N} [\mathbf{\mu}(t_0)] \} \\
\mathbf{\mu}(t_{h+1}) & = f_n \{ \mathbf{N}_{n[\mathbf{\mu}(t_h), \pi(A, t_h)]} \} \\
\end{align*}
\]

where:

- \( N \rightarrow \) number of agents,
- \( \mathbf{\mu}(t_h) \rightarrow \) parameter of improvement,
- \( \pi(A, t) \rightarrow \) parameter of progress.

There are three fundamental types of systems of improvement and progress, and for each we can assume irreversibility, strong reversibility and weak reversibility.

---

10 These systems are examples of the mechanism of increasing returns in collective phenomena (Arthur, 1994).
a) **MEDIAL SYSTEMS**, whose model is:

\[
\begin{align*}
\mu_i(t_0) & \leftarrow \langle \text{CHANCE/PROGRAMME} \rangle \quad 1 \leq i \leq N \\
\pi(A, t_h) & = \frac{1}{N} \sum_{1 \leq i \leq N} \mu_i(t_h) \quad h = 0, 1, 2, \ldots \\
\mu_i(t_{h+1}) & = \{ \mu_i(t_h) + p_i \Delta \mu_i(t_h) \} + \{ r_i [k \mu_i(t_h) + h \pi(A, t_h)] \} \quad 1 \leq i \leq N \\
\Delta \mu_i(t_h) & = \mu_i(t_h) - \pi(A, t_h) \quad 1 \leq i \leq N
\end{align*}
\]

where:
- \( \Delta \mu_i(t_h) = \mu_i(t_h) - \pi(A, t_h) \), denotes the *deviation* between the *individual improvement* level and the mean level denoting *collective progress*;
- \( p_i \) and \( r_i \) are measures of probability that represent the necessitating factors and may present different values according to the sign of \( \Delta \mu(n, t) \);
- \( i_i \) is a random coefficient.
- \( k \) and \( h \) are scalar coefficients, but we may normally assume that \( h=0 \).

The model shows how micro behaviours aim at reaching and/or exceeding a parameter of progress which represents an *AVERAGE* (other forms are admitted) of the measures of the parameter of improvement in the base agents.

The macro behaviour of the system leads to a continual readjustment of the average, so that the individual improvement leads to an advancement in the average progress, which, in turn, gives a boost to individual improvement.

b) **MAXIMAL SYSTEMS**, or systems of *pursuit*, whose model is:

\[
\begin{align*}
\mu_i(t_0) & \leftarrow \langle \text{CHANCE/PROGRAMME} \rangle \quad 1 \leq i \leq N \\
\pi(A, t_h) & = \text{Max} \{ \mu_i(t_h) \} = \mu_M(t_h) \quad h = 0, 1, 2, \ldots \\
\mu_i(t_{h+1}) & = \{ \mu_i(t_h) + p_i \Delta \mu_i(t_h) \} + \{ r_i [k \mu_i(t_h) + h \pi(A, t_h)] \} \quad 1 \leq i \leq N \\
\Delta \mu_i(t_h) & = \mu_i(t_h) - \mu_M(t_h) \quad 1 \leq i \leq N
\end{align*}
\]

where \( \Delta \mu_i(t_h) = \mu_i(t_h) - \mu_M(t_h) \) represents the *quantum of inferiority* perceived by each Agent compared with the improvement parameter of the *leader agent*.

The model shows that the parameter of progress is represented by the maximum value assumed by the parameters of improvement which characterize the agents of the system (the agent to which this value belongs is referred to as “the best”).

All the other agents thus present a state which is inferior to the best and try to improve for their part; the agent that succeeds in being the best becomes the guide for progress and gives a push toward further improvement.
We thus witness micro behaviours aimed at reducing the inferiority with respect to the level of progress, and this causes a macro behaviour whose effect is to raise the average level of improvement, so that some agents manage to further raise the previous level of progress.

**MINIMAL SYSTEMS**, or systems of flight, whose model is:

\[
\begin{align*}
\mu_i(t_0) &\leftarrow \text{« CHANCE/PROGRAMME »} & 1 \leq i \leq N \\
\pi(A, t_h) &= \text{Min}_i \mu_i(t_h) = \mu_m(t_h) & h=0, 1, 2, \ldots \\
\mu_i(t_{h+1}) &= \{ \mu_i(t_h) + p_i \Delta \mu_i(t_h) \} + \{ r_i [k \mu_i(t_h) + h \pi(A, t_h)] \} & 1 \leq i \leq N \\
\Delta \mu_i(t_h) &= \mu_i(t_h) - \mu_m(t_h) & 1 \leq i \leq N
\end{align*}
\]

where \(\Delta \mu_i(t_h) = \mu_i(t_h) - \mu_m(t_h)\) represents the *quantum of superiority* perceived by each Agent compared with the improvement parameter of the *base agent*.

These systems act in a symmetrical way with respect to the previous ones, since the parameter of progress is represented by the minimum level reached by the improvement parameter; all the other micro behaviours are thus superior. Each agent of the system tries to outdistance as much as possible its own level of improvement from the level of progress, to flee from the minimum level of improvement, to increment its own superiority. This leads to a general increase in the average level of improvement, which ends up raising the parameter of progress, further boosting the levels of improvement.

Let us assume a system of ten agents described by Figure 4.(A) shows the dynamics of this system under different hypotheses of reversibility.

As we can easily note, if both \(p_i\) and \(r_i\) admit reversibility, then the system is *strong reversible*; if only one of the two probabilities admits reversibility (generally \(r_i\)), the system is *weak reversible*; elsewhere it is *irreversible* and improvement and progress are continuously increasing. The more reversibility is introduced the more the macro and micro behaviours are chaotic, as we can verify by simulating dynamics for 20 iterations (to simulate, go to figure 4).
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